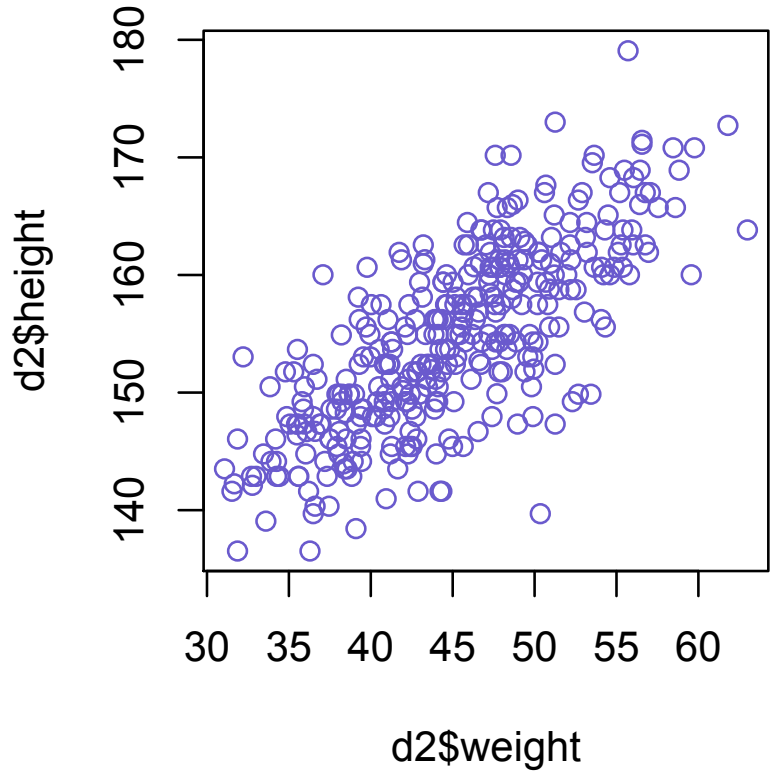
Adding a predictor variable

- How does weight describe height?



Adding a predictor variable

- Use a linear model of the mean, *mu*:

$$h_i \sim \text{Normal}(\mu_i, \sigma) \qquad \text{[likelihood]}$$

$$\mu_i = \alpha + \beta(x_i - \bar{x}) \qquad \text{[linear model]}$$

$$\alpha \sim \text{Normal}(178, 20) \qquad [\alpha \text{ prior}]$$

$$\beta \sim \text{Normal}(0, 10) \qquad [\beta \text{ prior}]$$

$$\sigma \sim \text{Uniform}(0, 50) \qquad [\sigma \text{ prior}]$$

Adding a predictor variable

$$h_i \sim \text{Normal}(\mu_i, \sigma)$$

$$\mu_i = \alpha + \beta(x_i - \bar{x})$$

mean on row i

mean when $x_i = 0$
"intercept"

mean weight

weight on row i

change in mean, per unit change x_i
"slope"

Prior predictive distribution

- What do these priors mean?
- Let's simulate to find out!

$$h_i \sim \text{Normal}(\mu_i, \sigma) \qquad \text{[likelihood]}$$

$$\mu_i = \alpha + \beta(x_i - \bar{x}) \qquad \text{[linear model]}$$

$$\alpha \sim \text{Normal}(178, 20) \qquad [\alpha \text{ prior}]$$

$$\beta \sim \text{Normal}(0, 10) \qquad [\beta \text{ prior}]$$

$$\sigma \sim \text{Uniform}(0, 50) \qquad [\sigma \text{ prior}]$$

Prior predictive distribution

```
set.seed(2971)
N <- 100                    # 100 lines
a <- rnorm( N , 178 , 20 )
b <- rnorm( N , 0 , 10 )
```

R code 4.38

$$\alpha \sim \text{Normal}(178, 20)$$

$$\beta \sim \text{Normal}(0, 10)$$

Prior predictive distribution

```
set.seed(2971)
N <- 100                    # 100 lines
a <- rnorm( N , 178 , 20 )
b <- rnorm( N , 0 , 10 )
```

$$\alpha \sim \text{Normal}(178, 20)$$
$$\beta \sim \text{Normal}(0, 10)$$

R code 4.38

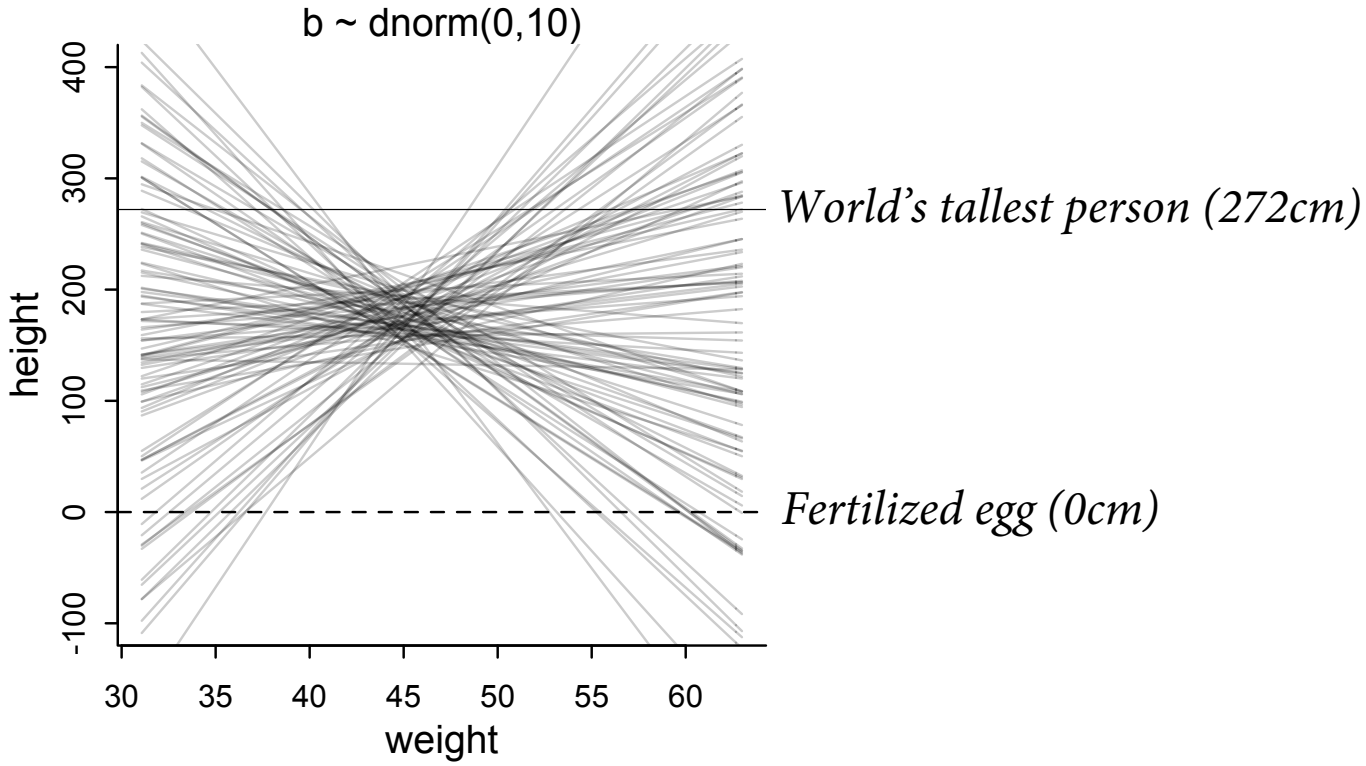


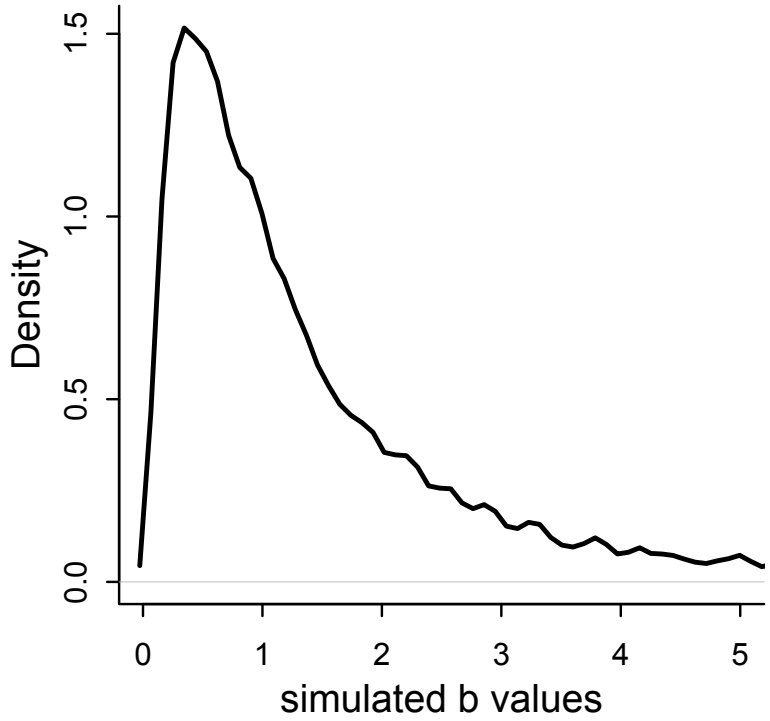
Figure 4.5

Prior predictive distribution

R code 4.40

```
b <- rlnorm( 1e4 , 0 , 1 )
dens( b , xlim=c(0,5) , adj=0.1 )
```

$$\beta \sim \text{Log-Normal}(0, 1)$$



Prior predictive distribution

R code 4.41

```
set.seed(2971)
N <- 100                    # 100 lines
a <- rnorm( N , 178 , 20 )
b <- rlnorm( N , 0 , 1 )
```

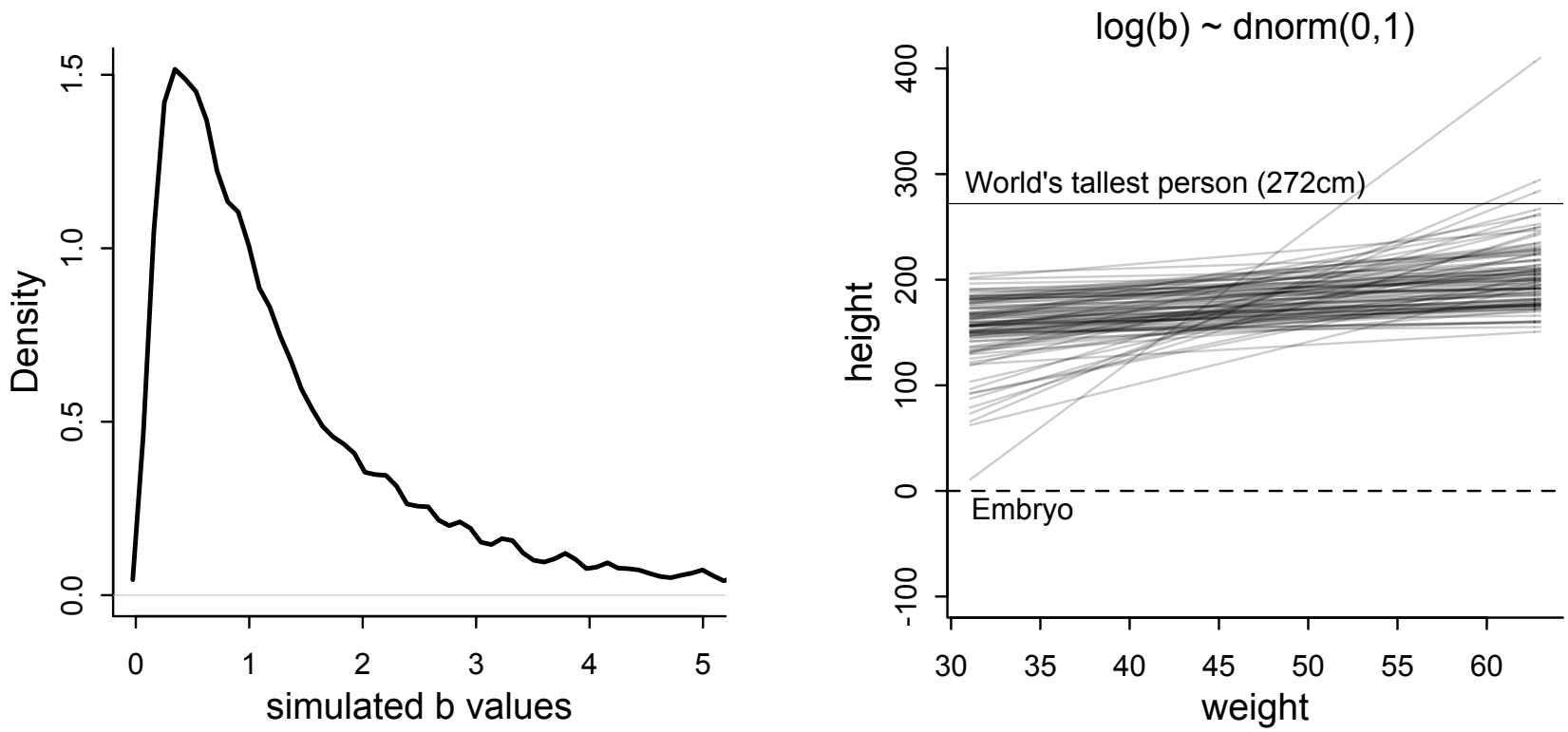


Figure 4.5

Approximate the posterior

R code 4.42

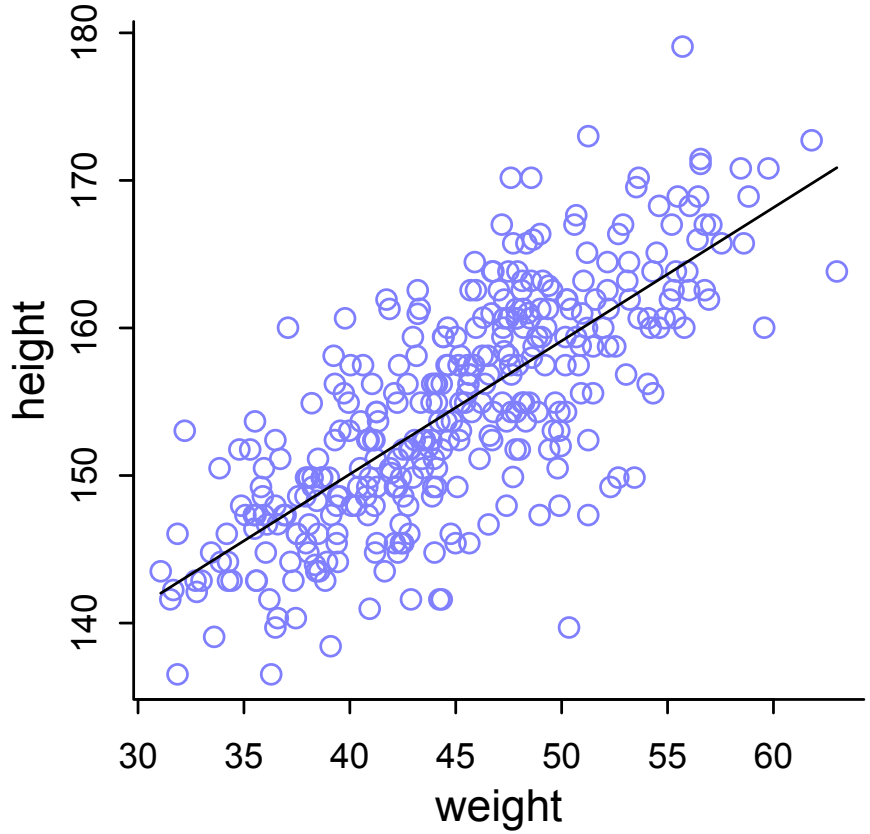
```
# load data again, since it's a long way back
library(rethinking)
data(Howell1)
d <- Howell1
d2 <- d[ d$age >= 18 , ]

# define the average weight, x-bar
xbar <- mean(d2$weight)

# fit model
m4.3 <- quap(
    alist(
        height ~ dnorm( mu , sigma ) ,
        mu <- a + b*( weight - xbar ) ,
        a ~ dnorm( 178 , 20 ) ,
        b ~ dlnorm( 0 , 1 ) ,
        sigma ~ dunif( 0 , 50 )
    ) ,
    data=d2 )
```

$$h_i \sim \text{Normal}(\mu_i, \sigma)$$
$$\mu_i = \alpha + \beta(x_i - \bar{x})$$
$$\alpha \sim \text{Normal}(178, 20)$$
$$\beta \sim \text{Log-Normal}(0, 1)$$
$$\sigma \sim \text{Uniform}(0, 50)$$

R code 4.44

```
precis( m4.3 )
```

```
        mean   sd   5.5%  94.5%
a     154.60 0.27 154.17 155.03
b       0.90 0.04   0.84   0.97
sigma   5.07 0.19   4.77   5.38
```



R code 4.46

```
plot( height ~ weight , data=d2 , col=rangi2 )
post <- extract.samples( m4.3 )
a_map <- mean(post$a)
b_map <- mean(post$b)
curve( a_map + b_map*(x - xbar) , add=TRUE )
```

Figure 4.6

Sampling from the posterior

- Want to get uncertainty onto that graph
- Again, sample from posterior
  1. Use mean and standard deviation to approximate posterior
  2. Sample from *multivariate normal* distribution of parameters
  3. Use samples to generate predictions that "integrate over" the uncertainty

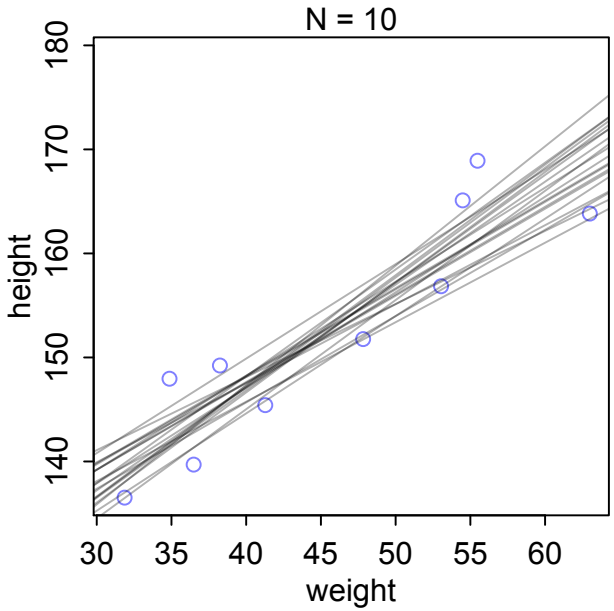
Sampling from the posterior

R code 4.47

```
post <- extract.samples( m4.3 )
post[1:5,]
```

```
         a         b    sigma
1 154.5505 0.9222372 5.188631
2 154.4965 0.9286227 5.278370
3 154.4794 0.9490329 4.937513
4 155.2289 0.9252048 4.869807
5 154.9545 0.8192535 5.063672
```

Posterior is full of lines



R code 4.47

```
post <- extract.samples( m4.3 )
post[1:5,]
```

```
          a         b    sigma
1 154.5505 0.9222372 5.188631
2 154.4965 0.9286227 5.278370
3 154.4794 0.9490329 4.937513
4 155.2289 0.9252048 4.869807
5 154.9545 0.8192535 5.063672
```

Figure 4.7

Posterior is full of lines

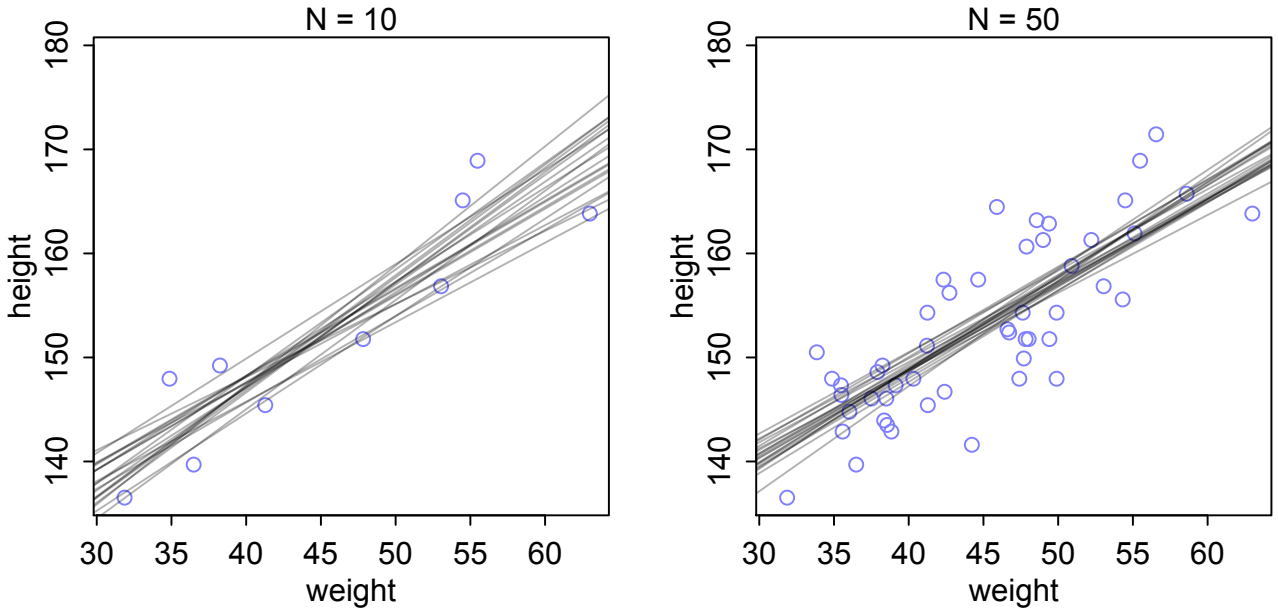


Figure 4.7

Posterior is full of lines

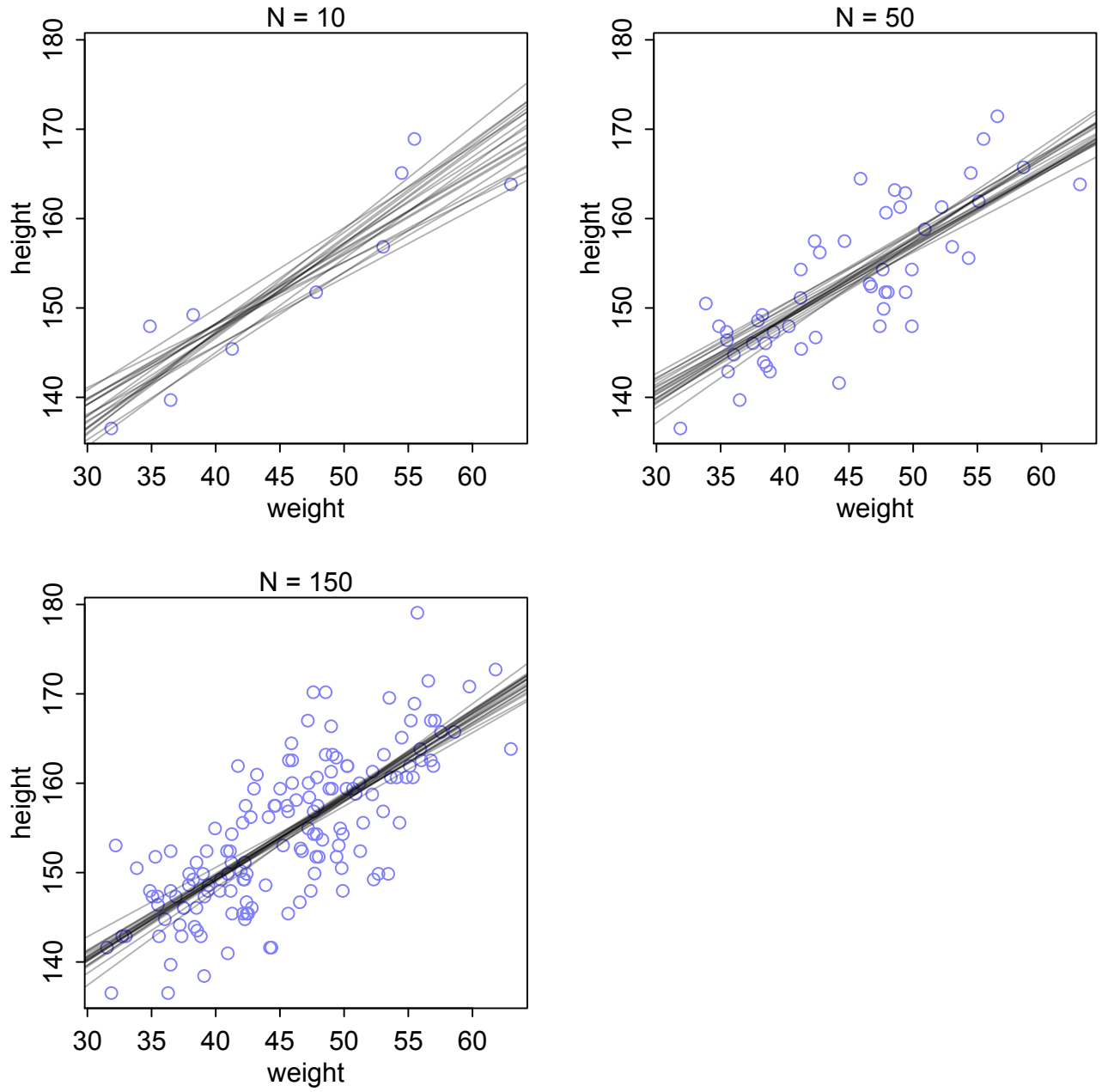


Figure 4.7

Posterior is full of lines

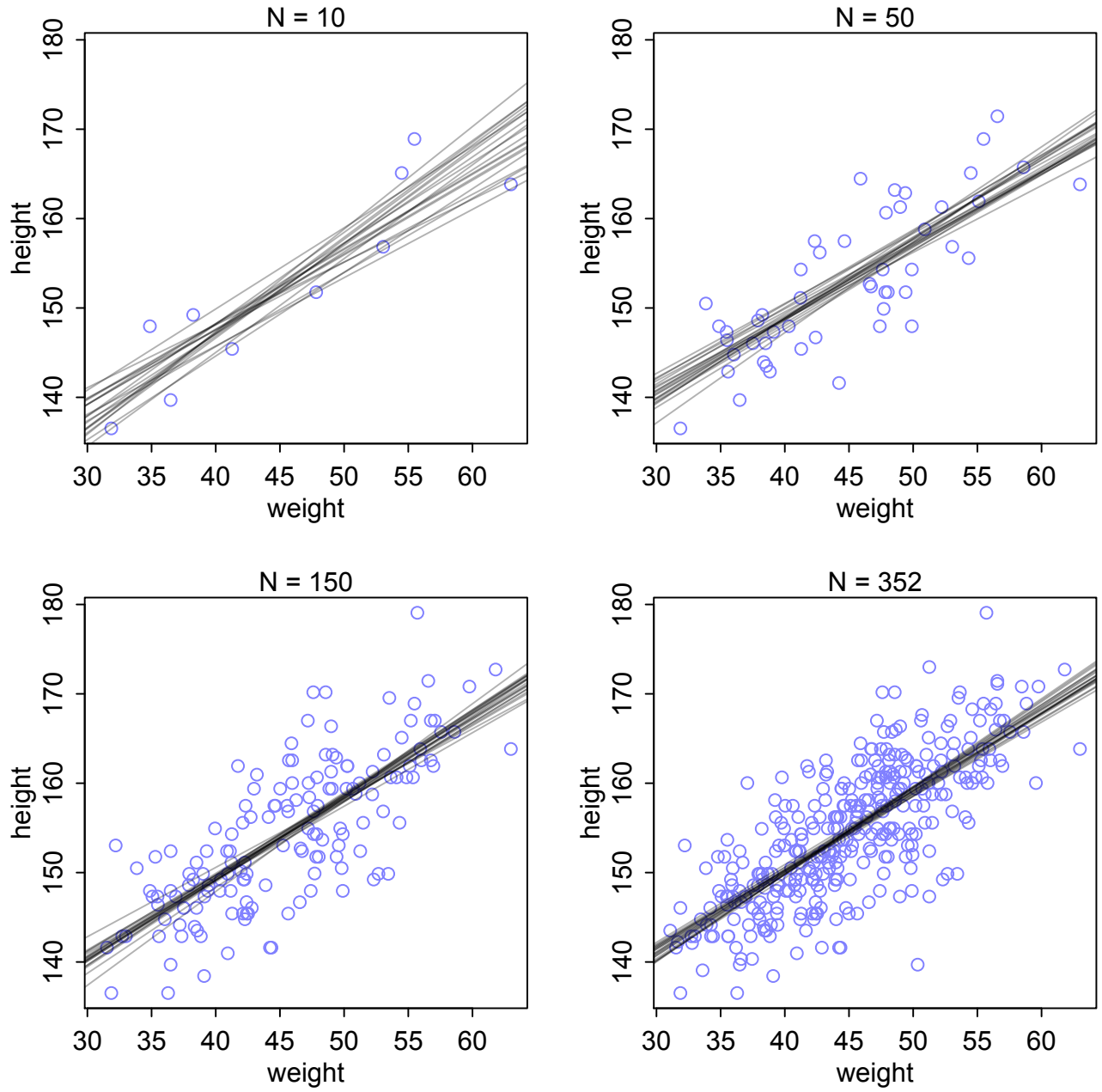


Figure 4.7